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DYNAMICS OF MAGNETOPHORETIC SEPARATION OF A SUSPENSION OF SLIGHTLY MAGNETIC MICROPARTICLES IN A HIGH-GRADIENT FIELD OF A MAGNETIZED ROD

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A macroscopic model of the process of magnetophoretic separation of slightly magnetic particles from a suspension under the action of a high-gradient magnetic field with account for the particle size is proposed. The mechanisms of separation of dia- and paramagnetic particles from a narrow fluid layer in an inhomogeneous field of a magnetized rod have been investigated.

Keywords: *magnetophoresis, high-gradient magnetic separation, computational modeling.*

Introduction. The development and introduction of medical technologies connected with the transplantation of cells require the elaboration of methods for cell sorting. The existing methods are based on the discrimination of cells by the specific weight and immune markers on the surface. Their application involves multistage manipulations, requires expensive preparations, and is accompanied by considerable losses and a decrease in the vital activity of functional cells. Additional possibilities of sorting cells are provided by the difference in their magnetic properties. Interest in magnetic separation of cells arose in the 1970s [1] due to the fact that the magnetic field combining a high intensity with a strong small-scale inhomogeneity could be used to separate from a fluid very small slightly magnetic particles including particles of cell suspensions. Since that time the main objects of magnetophoretic investigations have been the component elements of blood, especially erythrocytes whose magnetic properties are highly sensitive to a change in the state of the heme molecule in redox reactions [2–4]. In [5], the possibility of magnetic enrichment and cleaning of a suspension of insulin-producing cells to be transplanted to patients with diabetes mellitus was shown. The development of a technology of magnetic sorting of cells and other slightly particles calls for the solution of a complex of scientific and engineering problems. Earlier we considered methods for calculating the structure of the magnetic field of filters [6–8] and methods for investigating the magnetophoretic behavior and magnetic properties of individual slightly magnetic particles and cells [5, 9, 10]. An important problem for the technology of magnetophoretic separation is the investigation of the macroscopic effect of separation of particles from the suspension volume. The aim of the present work is to construct a mathematical model and investigate numerically the process of separation of slightly magnetic particles from the suspension filling the vertical flat channel under the action of a high-gradient magnetic field creating an electrodynamic force in the channel plane across the gravity force.

Geometry and Formulation of the Problem. The geometry of the problem under consideration is given in Fig. 1. The suspension with particles is in the narrow slit channel 1 whose face is touched by a ferromagnetic rod 2 of rectangular cross-section of thickness $2a$ exceeding 2–3 times the thickness of the channel. The length of the rod C is much larger than its two other dimensions. The external homogeneous field $\mathbf{H}_0$ is applied across the rod in the channel plane and magnetizes the rod to saturation. Associated with the channel is the Cartesian coordinate system X, Y, Z. Let us formulate a model of the behavior of individual particles. Under the action of the magnetic force created by the inhomogeneous field of the rod the horizontal motion of particles in the gravitational field is superimposed on the vertical motion. Its description is based on the macroscopic approach and presupposes a uniform distribution of magnetization in the volume of a particle (cell), which is fully acceptable in studying its mechanical behavior as a whole. According to the electrodynamics of continua [11], a body of volume V having magnetic susceptibility χ and placed into a medium with susceptibility χ_0 in an inhomogeneous magnetic field $\mathbf{H}$ whose scale of inhomogeneity is large compared to the dimensions of the body is subjected to the electrodynamic force

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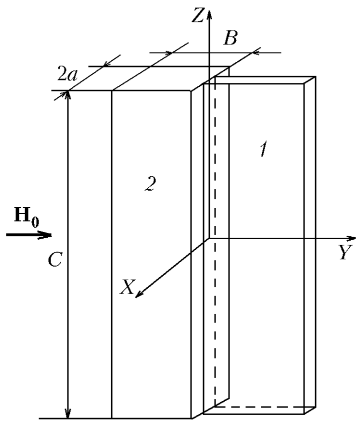


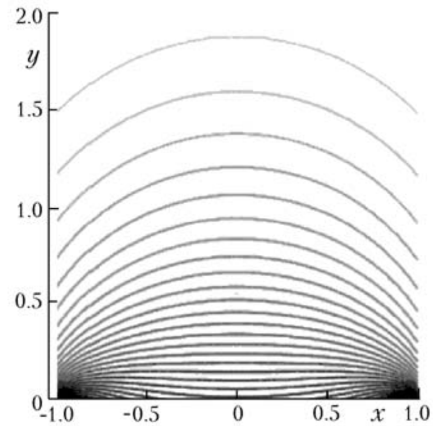


Fig. 1. Scheme of the problem: 1) channel; 2) ferromagnetic rod.

Fig. 2. Isolines of the magnetophoretic potential.

$$\mathbf{F}_{\mathrm{m}} = \frac{1}{2}\Delta\chi V \nabla \mathbf{H}^2 , \quad \Delta\chi = \chi - \chi_0 . \tag{1}$$

The magnetic field is given by the sum of the external field and the self-field of the magnetized rod, $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}'$. Using the half-thickness of the rod a as a scale distance and the quantity $2\pi I_{\mathrm{s}}$, where I_{s} is the magnetization of the rod saturation, as a field scale, we write the distribution of the self-field intensity in the form [10]

$$h_x(x, y) = \frac{H'_x}{2\pi I_{\mathrm{s}}} = \frac{1}{2\pi} \ln \frac{\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + (y+b)^2\right]}{\left[(x-1)^2 + y^2\right]\left[(x+1)^2 + (y+b)^2\right]} ,$$

$$h_y(x, y) = \frac{H'_y}{2\pi I_{\mathrm{s}}} = \frac{1}{\pi}\left[\arctan\frac{1-x}{y} + \arctan\frac{1+x}{y} - \arctan\frac{1-x}{y+b} - \arctan\frac{1+x}{y+b}\right]. \tag{2}$$

Here $x = X/a$, $y = Y/a$, $b = B/a$. Taking into account the homogeneity of the external field, let us reduce the expression for the magnetic force (1) to the form

$$\mathbf{F}_{\mathrm{m}} = -\nabla\Phi , \quad \Phi = -2\Delta\chi V (\pi I_{\mathrm{s}})^2 \left[\mathbf{h}^2 + \frac{H_0}{\pi I_{\mathrm{s}}}\mathbf{eh}\right], \tag{3}$$

where the magnetophoretic potential Φ was introduced. Choosing the quantity $\Phi^* = 2\Delta\chi V(\pi I_{\mathrm{s}})^2$ as a scale of magnetophoretic potential, let us write the dimensionless potential

$$\varphi = \frac{\Phi}{\Phi^*} , \tag{4}$$

where $\varphi = -\mathbf{h}^2 = P\mathbf{eh}$, $P = H_0/(\pi I_{\mathrm{s}})$.

The thus dedimensionalized magnetophoretic potential pertains to paramagnetic particles and that taken with the opposite sign pertains to diamagnetic particles. In so doing, paramagnetic particles move in the direction of the minimum of the potential Φ and diamagnetic ones move in the direction of its maximum. The isolines of the magnetophoretic potential in the band $-1 < x < 1$, $y > 0$, for typical conditions a = 0.2 mm, B = 5 mm, H_0 = 9 kOe, I_{s} = 1700 G (P = 1.7) are given in Fig. 2. It is seen that in the middle part of the band the potential changes slightly in the transverse direction. If the slit channel adjoining the rod has a thickness less than half of the rod thickness, then the motion of particles across the channel can be neglected.

Let us write the equation of motion of a particle in the inertialess approximation from the condition of mutual compensation of the magnetic, sedimentation, and viscous forces:

$$\mathbf{v} \equiv \frac{d\mathbf{R}}{dt} = \alpha \mathbf{F}\,, \quad \mathbf{F} = \mathbf{F}_{\mathrm{m}} + \mathbf{g}\Delta\rho V\,. \tag{5}$$

We obtain the equation describing the change in the particle density in the channel with the time of action of the field from the law of conservation of the number of particles:

$$\frac{\partial n}{\partial t} = -\operatorname{div} \mathbf{J}\,, \tag{6}$$

where $\mathbf{J} = n\cdot\mathbf{v}$ is the particle flux, n is the numerical concentration of particles, and $\mathbf{v}$ is the velocity defined by Eq. (5). Note that the model under consideration does not take into account the Brownian thermal diffusion of particles. For the typical cell size $d_{\mathrm{c}} = 10$ μm, the fluid viscosity $\eta = 10^{-2}$ P, and the temperature $T = 300$ K, and the characteristic time of thermal diffusion $\tau_\beta = 3V\eta/(k_{\mathrm{B}}T) \approx 10^3$ sec. In this time the particle under the action of random force shifts for a distance of the order of its diameter. The model under consideration is applicable when the magnetophoresis rate is much higher than the estimated thermal diffusion rate. Note that if this condition is not fulfilled, then the use of the magnetophoretic method of separation of particles makes no sense.

To describe the process of accumulation of particles on the faces, it is necessary to take into account the decrease in their mobility with increasing concentration. Let us denote the highest possible concentration corresponding to the concentration in a freely sedimented layer by n^*. At this concentration the coefficient of mobility of particles goes to zero. It should be taken into account that the mobility of a particle in a strongly stratified suspension depends on the direction of motion. Consider a particle situated at the boundary of a completely deposited layer. Obviously, its mobility in the direction of the layer is equal to zero, and in the opposite direction it is comparable to the mobility of a free particle. Moreover, since the sizes of the considered region are comparable to the particle size, the finite size of particles should be taken into account in the macroscopic model. This can be done in the expression for the coefficient of mobility assuming that the mobility at a certain point is determined by the concentration existing at a distance of the particle diameter from the given point in the direction of the force acting on the particle. For example, the mobility of a particle in the direction of the deposited layer of particles is assumed to be equal to zero at a distance of the diameter from the boundary layer formed by the centers of particles on its surface. Thus, we find the coefficient of mobility of particles in the form

$$\alpha\,(\mathbf{R}) = \alpha_0 \left(1 - \frac{1}{n^*}\left[n\left(\mathbf{R} + d\,\frac{\mathbf{F}}{F}\right)\right]\right), \tag{7}$$

and the particle flux density is found in the form

$$\mathbf{J} = \alpha_0\, n\,(t, \mathbf{R})\, \mathbf{F}\left(1 - \frac{n\left(t, \mathbf{R} + d\,\frac{\mathbf{F}}{F}\right)}{n^*}\right). \tag{8}$$

Having introduced the relative volume concentration of particles $c = n/n^*$, we write the transfer equation in the following form:

$$\frac{\partial c}{\partial t} = -\operatorname{div} \mathbf{j}\,, \quad \mathbf{j} = \alpha_0 c\,(t, \mathbf{R})\, \mathbf{F}\left[1 - c\left(t, \mathbf{R} + d\,\frac{\mathbf{F}}{F}\right)\right]. \tag{9}$$

We assume that the channel under consideration has a larger length compared to the thickness and the width and the sedimentation rate of particles is small, so that at a distance from the upper and lower edges of the channel the concentration can be assumed to be independent of the vertical direction. We do not take into account the motion of par-

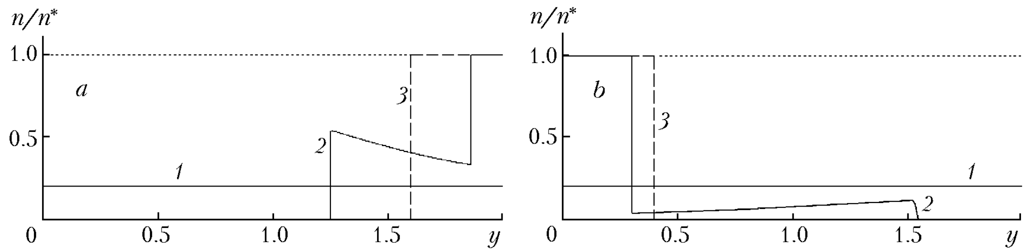


Fig. 3. Change in the concentration profile of diamagnetic (a) and paramagnetic (b) particles depending on the time of exposure to the field: 1) $\tau = 0$; 2) $\tau_0/2$; 3) τ_0.

ticles across the channel (along the OX axis), since the magnetic force acting in this direction, as was shown above, is small. Thus, we arrive at a one-dimensional problem. Using the distance scale a, the magnetic force scale $F^* = |\Delta\chi| V(2\pi I_s)^2/(2a)$, and the time scale $t^* = 2a^2[\alpha_0 |\Delta\chi| V(2\pi I_s^2)]^{-1}$, we reduce Eq. (9) for this case to the dimensionless form

$$\frac{\partial c(\tau, y)}{\partial \tau} + \frac{\partial j}{\partial y} = 0 , \quad j = c(\tau, y) f [1 - c(\tau, y - d \operatorname{sign}(\Delta\chi))] . \tag{10}$$

Here $\tau = t/t^*$; $y = Y/a$; $f = F_{my}|_{x=0}/F^* = -\partial\varphi(0, y)/\partial y$; it was taken into account that the direction of the magnetic force is determined by the sign of $\Delta\chi$.

Let us formulate the initial and boundary conditions. We assume that in the initial state the particles are uniformly distributed in the cell volume: $n(0, y) = n_0$, and at the boundaries the impermeability condition $j|_{y=0} = 0$, $j|_{y=L} = 0$ is fulfilled.

Finite-Difference Scheme. Let us formulate the finite-difference approximation of the above system of equations, for which we introduce the space-time grid (τ_j, y_j). The number of discretization intervals along the OX axis is equal to N, the step on the coordinate $s_y = L/N$, and the time step $s_\tau = s_y K$. Note that the choice of the space step in the considered problem presents a certain methodological problem because of the existence of a sharp boundary of the deposited layer of particles. On the one hand, the macroscopic approach presupposes consideration of elementary volumes containing a large number of particles. On the other hand, the account of the particle size used here in the expression for the coefficient of mobility presupposes the knowledge of the concentration with a step equal to the particle diameter. Choosing a step equal to the diameter, we can satisfy both requirements, since the considered flat layer of thickness equal to the diameter contains a large number of particles.

Consequently, giving the initial relative concentration of particles c_0 and the sign of the difference between the susceptibilities of the particles and the fluid sign ($\Delta\chi$) = ± 1, we write the finite-difference algorithm for calculating the change in the concentration:

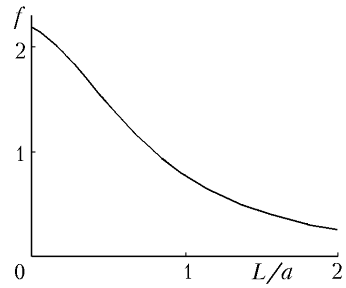
$$c_{0,k} = c_0 , \quad k = 0, 1, ..., N ;$$

$$c_{i+1,k} = c_{i,k} + \frac{K}{2}(j_{i,k-1} - j_{i,k+1}) , \quad j_{i,k} = c_{i,k} f_k [1 - c_{i,k-\mathrm{sign}(\Delta\chi)}] , \quad k = 1, 2, ..., N-1 ;$$

$$c_{i+1,0} = c_{i,0} - Kj_{i,1} , \quad c_{i+1,N} = c_{i,N} + Kj_{i,N-1} , \qquad j_{i,0} = j_{i,N} = 0 .$$

Results of the Calculations. Consider the behavior of suspensions of dia- and paramagnetic particles under the action of the magnetic field of a rod with the above characteristics.

The change with time in the concentration profiles in the suspensions of dia- and paramagnetic particles calculated for $c_0 = 0.2$ and $N = 2000$ is shown in Fig. 3. As is seen, in the process of separation of particles in the layer three regions are formed: the sediment region ($c = 1$), the region of purified fluid ($c = 0$), and the intermediate region. The fundamental difference between dia- and paramagnetic particles is that the particle concentration in the intermediate region in the first case is larger than the initial concentration c_0, and in the second case it is smaller. The width



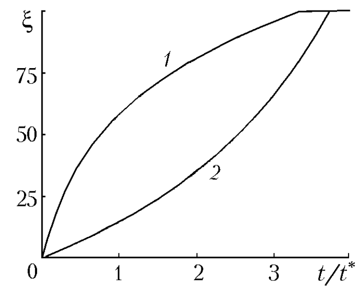


Fig. 4. Dimensionless magnetic force as a function of the dimensionless distance to the rod.

Fig. 5. Time dependence of the degree of sedimentation for paramagnetic (1) and diamagnetic (2) particles. ξ, %.

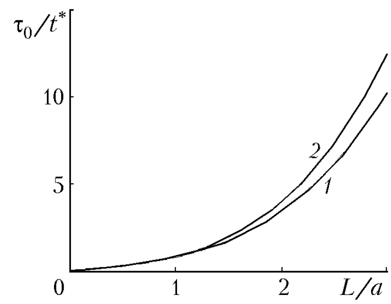


Fig. 6. Dependence of the time of complete separation of paramagnetic (1) and diamagnetic (2) particles on the channel width.

of the layer of the fluid cleaned from diamagnetic particles increases with time faster than that of the fluid cleaned from paramagnetic particles. The nature of such behavior is due to the character of the change in the magnetic force in the layer (see Fig. 4). Let us introduce the degree of sedimentation ξ as a ratio of the number of separated particles to their total number: $\xi = H_s/(c_0 L)$. Here H_s is the width of the sediment layer; L is the width of the channel (in units of a). The time dependences of the degree of sedimentation of paramagnetic and diamagnetic particles are given in Fig. 5. They have, respectively, a convex and a concave character, i.e., the sedimentation of paramagnetic particles is more intensive at the initial stage of the process, and that of diamagnetic particles — at the final stages. The dependence of the time of complete separation of paramagnetic and diamagnetic particles on the channel width is given in Fig. 6. As is seen, the sedimentation of diamagnetic particles requires more time.

Conclusions. The results of the numerical investigations agree with the physical notions about the character of magnetophoresis in dia- and paramagnetic suspensions, which makes it possible to draw the conclusion that the proposed model taking into account the finite size of particles can be used to study the macroscopic effect of separation and optimize the geometric characteristics and operational conditions of high-gradient magnetic filters depending on the properties of separated suspensions and requirements on separation results.

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NOTATION

a, half-thickness of the magnetic rod, cm; B, width of the magnetic rod, cm; b, dimensionless width of the magnetic rod; **e**, unit vector in the direction of the external magnetic field; c, dimensionless concentration; c_0, initial dimensionless concentration; d, particle diameter, cm; f, dimensionless magnetophoretic force; F^*, force scale, dyn; F_m, magnetophoretic force, dyn; **g**, vector of gravitational acceleration, cm/sec^2; **H**, magnetic field, Oe; **H′**, self-magnetic field of the rod, Oe; $\mathbf{H}_0$, external homogeneous magnetic field, Oe; **h**, dimensional self-magnetic field intensity; h_x, h_y, components of the dimensional vector of magnetic field intensity; I_s, saturation magnetization of the magnetic rod, G; **j**, particle flux, cm^{-2}·sec^{-1}; K, coefficient relating the time and coordinate steps; k_B, Boltzmann constant; L, channel width, dimensionless quantity; N, number of discretization intervals; n, numerical concentration, cm^{-3}; n^*, highest possible concentration in the sedimentation layer, cm^{-3}; P, dimensionless intensity of external field; $\mathbf{R}$, radius vector, cm; s_y, coordinate step; s_τ, time steps; t, time, sec; T, temperature, K; t^*, time scale, sec; V, particle volume, cm^3; $\mathbf{v}$, particle velocity, cm/sec; X, Y, Z, Cartesian coordinates, cm; x, y, dimensionless Cartesian coordinates; α, coefficient of mobility of particles, $sec \cdot g^{-1}$; α_0, coefficient of mobility of a solitary particle in unlimited volume of fluid, $sec \cdot g^{-1}$; $\Delta\rho$, density difference between the particle and the fluid, $g \cdot cm^{-3}$; $\Delta\chi$, magnetic susceptibility difference between the particle and the fluid; χ, magnetic susceptibility of particles; χ_0, magnetic susceptibility of the carrier medium; η, viscous friction coefficient, P; ξ, degree of sedimentation of particles; τ, dimensionless time; Φ, magnetophoretic potential, Erg; Φ^*, scale of magnetophoretic potential, Erg; φ, dimensionless magnetophoretic potential. Subscripts: s, saturation; m, magnetophoretic; c, cell.

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